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Pure rank preferences and variation in risk-taking behavior

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ABSTRACT

Assuming that an individual's rank in the wealth distribution is the only factor determining the individual's wellbeing, we analyze the individual's risk preferences in relation to gaining or losing rank, rather than the individual's risk preferences towards gaining or losing absolute wealth. We show that in this characterization of preferences, a high-ranked individual is more willing than a low-ranked individual to take risks that can provide him with a rise in rank: relative risk aversion with respect to rank in the wealth distribution is a decreasing function of rank. This result is robust to incorporating (the level of) absolute wealth in the individual's utility function.

"The desire of ... obtaining ... rank among our equals, is, perhaps, the strongest of all our desires" (Smith, 1759, Part VI, Section I, Paragraph 4).

1. Introduction

Imagine that individuals care so much about their rank in the wealth distribution that they set aside other variables; it is as if individuals care only about their rank. This characterization can be explained, for example, by cultural traits after basic needs are already catered for, or as the preferences of the wealthy.[1] Support for such a stance comes from evidence which is in congruence with the Easterlin (1974) paradox provided, for example, by Clark and Oswald (1996), and Frank (1997). We study one of the configurations alluded to by Weiss and Fershtman (1998), who wondered whether the rank or status of an individual should be included in the individual's utility function directly as a "good," or whether high rank should be treated as a means of obtaining tangible goods and services from which utility is to be derived. (Consult also the related review of empirical and experimental findings by Heffetz and Frank, 2011.) How will the readiness to take risks of individuals who care only about their rank change when their rank changes?

While there is some acknowledgement in the received literature of a link between relative wealth (status) and risk-taking (gambling) behavior, the received writings do not follow the track that we pursue in this paper. Most notably, Gregory (1980) and Robson (1992) refer to a link between relative wealth and gambling behavior, remarking that the incorporation of relative wealth (status) in an individual's utility function can explain the Friedman and Savage (1948) paradox of seemingly inconsistent risk-taking behavior of an individual following a change in his wealth. However, Gregory does not specify a link between relative wealth and any concrete measure of risk aversion. Robson investigates connections between wealth distributions and fair gambles, yet he too does not link status with any measure of risk aversion. Although Robson expands the individual's utility function to include a status term based on the individual's rank in the wealth distribution, he does not revise the utility function to include the individual's rank in the wealth distribution as the sole argument of the function.

In a way, we are building on the preceding considerations in that we focus on rank as the main determinant of the wellbeing

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[1] In a concert by the Three Tenors organized at Wembley Stadium, Pavarotti reputedly did not care how much he was paid so long as it was one pound more than Domingo. (The source of this story is not known to us.) An inference that can be drawn from this episode is that Pavarotti would presumably have cared more about his fee had he been placed far lower in the earnings scale, but once at the top, rank came to the fore.

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of an individual, and we analyze the individual's risk preferences towards gaining or losing (a wealth-conferred) rank, rather than the risk preferences towards gaining or losing absolute wealth. We find that under this characterization of risk preferences, a high-ranked individual is more willing to take risks that can confer a rise in rank than a low-ranked individual.

Stark (2019) explores the link between concern over having low relative wealth, actual wealth, and risk aversion. Specifically, Stark studies the relative risk aversion of an individual with particular social preferences: his wellbeing is influenced by his relative wealth, and by how concerned he is about having low relative wealth. The core assumption of the study, which is supported by empirical evidence, is that individuals who are positioned higher in the wealth hierarchy care more about relative wealth than individuals who are positioned lower down. Holding constant the individual's absolute wealth, two results are obtained. First, if the individual's level of concern about low relative wealth does not change, the individual becomes *more* risk averse when he rises in the wealth hierarchy. Second, if the individual's level of concern about low relative wealth intensifies when he rises in the wealth hierarchy and if, in a precise sense, this intensification is strong enough, then the individual becomes *less* risk averse: the individual's desire to advance further in the wealth hierarchy is more important to him than possibly missing out on a higher rank. In a way, the present paper can be conceptualized as the limit of the Stark's study: not only is concern about low relative wealth - in this case rank - held constant; it is assumed to be so powerful that it completely "takes over" the individual's utility.

2. Pure rank preferences

We can start with an *intuitive* representation. Let there be a population of n individuals where n is a natural number bigger than 1. Let i be the individual's position on the ladder of ranks, $i=n,n-1,n-2,\dots,1$. We refer to the rank of an individual as the individual's name. Thus, individual n is the top-ranked individual, individual $n-1$ is the second-ranked individual, and so on, and individual 1 is the lowest-ranked individual. Let the utility of individual i be defined as

$$U(i)\equiv(1+n-i)^{-1}. \tag{1}$$

Thus, utility is measured by the inverse of a rank-based distance from the top rank (which is occupied by individual n), augmented by one so that the utility level of the top-ranked individual - a utility level of one - can serve as a baseline value. Essentially, utility is a measure of "rank deprivation." For example, when there are three individuals, 3, 2, and 1, individual 3 holds the top rank, he experiences no rank deprivation, and his utility, which is the maximal utility in the three-person population, is equal to $U(3)=(4-3)^{-1}=1$. Individual 2 is at a distance of one from the top rank, so he experiences rank deprivation, and his utility, which is lower than that of individual 3, is equal to $U(2)=(4-2)^{-1}=\frac{1}{2}$. Individual 1 is at a distance of two from the top rank and thus, out of the three individuals, he experiences the highest level of rank deprivation and his level of utility, which is the lowest, is equal to $U(1)=(4-1)^{-1}=\frac{1}{3}$.

If we forget temporarily that differentiation with respect to discrete i is sinful, we will have from (1) that

$$U'(i)=(1+n-i)^{-2},$$

$$U''(i)=2(1+n-i)^{-3}.$$

To assess how the willingness of an individual to take risks responds to a change in the individual's rank, we employ the Arrow–Pratt index of relative risk aversion (Pratt, 1964; Arrow, 1965, 1970), $r(i)$, which is defined as

$$r(i)\equiv\frac{-iU''(i)}{U'(i)}.$$

For the utility formulation in (1), this index takes the form

$$r(i)=\frac{-i\cdot 2(1+n-i)^{-3}}{(1+n-i)^{-2}}=-2\frac{i}{1+n-i},$$

and, therefore,

$$r'(i)=-2\frac{1+n}{(1+n-i)^2}<0.$$

We see that when i is bigger (which is tantamount to experiencing lower rank deprivation - the distance from the top rank is shorter), the willingness to take risks is greater; higher-ranked individuals are less reluctant to take risks.[2]

A mathematically cleaner approach, which escapes the "sin" of differentiation with respect to a discrete variable, is as follows.

We consider a population of measure one in which we index the individuals by a number $\rho\in[0,1]$. We assume that the population is characterized by some continuous wealth distribution. Without loss of generality, we equate the index number of an individual with the fraction of those in the population whose levels of wealth are smaller than or equal to the level of wealth of the individual.[3] The index ρ then represents the individual's continuous rank measure, and $1-\rho$, which is the fraction of those in the population whose levels of wealth are higher than the level of wealth of the individual, represents the individual's continuous rank deprivation measure.[4] Rank-utility can then be characterized as

$$U(\rho)\equiv[1+(1-\rho)]^{-1}=(2-\rho)^{-1}, \tag{2}$$

yielding derivatives

$$U'(\rho)=(2-\rho)^{-2},$$

and

$$U''(\rho)=2(2-\rho)^{-3}.$$

In the case of (2), the index of relative risk aversion as a function of ρ, $r(\rho)$, is

$$r(\rho)=-\rho\frac{2(2-\rho)^{-3}}{(2-\rho)^{-2}}=-2\frac{\rho}{2-\rho}.$$

Therefore,

$$r'(\rho)=-4\frac{1}{(2-\rho)^2}<0.$$

Thus, when the fraction of the lower-ranked individuals is higher, which is tantamount to the reference individual occupying a higher rank and thereby experiencing lower rank deprivation, the

[2] This result also holds true for a somewhat more general representation in which the individual's utility is defined as

$$U(i)\equiv(1+n-i)^{-\alpha},$$

where $\alpha>0$. Then $r(i)=-(\alpha+1)\frac{i}{1+n-i}$, and $r'(i)=-(\alpha+1)\frac{1+n}{(1+n-i)^2}<0$.

[3] This characterization is tantamount to assuming that the wealth levels $w\in[w_{min},w_{max}]$ are distributed according to a continuous probability distribution with cumulative distribution function $F(w)$. In that case, we index the individuals not by their wealth levels but rather by the values $\rho=F(w)$.

[4] The measure $1-\rho$ is analogous to the fraction $\frac{n-i}{n}$ in the preceding discrete model of population. For example, in a discrete wealth distribution of five individuals where 5 is the top rank, 4 is the second high rank, 3 is the third rank, 2 is the last but one rank, and 1 is the lowest rank, the analogous ρ measure of individual 3 is $\frac{3}{5}$, and the analogous $1-\rho$ measure of this individual is $\frac{2}{5}$.

reluctance of the reference individual to take risks is lower; a higher-ranked individual is more willing to take risks.

More generally, let

$$U(\rho)\equiv(2-\rho)^{-\alpha}, \tag{3}$$

where $\alpha>0$. Then

$$U'(\rho)=\alpha(2-\rho)^{-1-\alpha},$$

and

$$U''(\rho)=\alpha(1+\alpha)(2-\rho)^{-2-\alpha}.$$

Thus, in the case of (3),

$$r(\rho)=-(1+\alpha)\frac{\rho}{2-\rho},$$

and, therefore,

$$r'(\rho)=-2\frac{1+\alpha}{(2-\rho)^2}<0.$$

Namely, a higher-ranked individual is deterred less from risk taking.

A natural question to ask is whether the results concerning the "pure rank" preferences are also valid when the sole appearance of rank is "contaminated" by the presence of absolute wealth in the individual's utility function. Let

$$U(\rho,w)\equiv w^{\beta}(2-\rho)^{-\beta}, \tag{4}$$

where $\beta>0$, namely the individual's utility depends positively on the level of his wealth w, and negatively on the fraction of the individuals who occupy ranks higher than his.[5] In the case of (4), the index of relative risk aversion (with respect to rank) is

$$r_{\rho}(\rho)=-(1+\beta)\frac{\rho}{2-\rho},$$

and, therefore, we obtain that

$$r'_{\rho}(\rho)=-2\frac{1+\beta}{(2-\rho)^2}<0.$$

Namely a higher-ranked individual is more willing to take risks.

3. Conclusions

There is a widespread perception that wealthier people are more inclined to take risks. Observations that an increase in wealth increases the fraction of wealth that is invested in risky holdings strengthen this perception. Nonetheless, as in many other spheres, transferring a link to a cause requires caution. This applies to the current setting if a gain in wealth is combined with a rise in wealth rank. We have shown how when rank is of great importance, people revise their risk-taking preferences in response to a change in their rank. Considering the relationship between rank and willingness to take risks under alternative representations of rank preferences, we found that when rank is higher the willingness to take risks is higher. This observation holds even when a pure rank-preference utility function is expanded to incorporate wealth as an argument, such that the function exhibits a preference for more wealth.

The standard attention paid in the received literature to a gain in wealth as the driver of a lesser reluctance to take risks may overestimate the role of the gain in wealth. Conversely, to assume that willingness to take risks will not change when wealth is held constant will be misleading if at the same time a gain in rank occurs.

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[5] With the utility formulation in (4), we get that

$$\frac{\partial^2 U(\rho,w)}{\partial w\partial\rho}=\beta^2(2-\rho)^{-1-\beta}w^{\beta-1}>0,$$

namely the marginal utility of wealth is an increasing function of the individual's rank. This attribute of the utility function is similar to the characterization of the utility schedule by Friedman and Savage (1948).